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Single Zero and Zero Counting Methods for Fuzzy Transportation Problems: A Comparative Zero-Based Optimisation Framework

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Abstract

This paper develops and compares two direct zero-based procedures for fuzzy transportation problems: the Single Zero Method (SZM) and the Zero Counting Method (ZCM). Generalised trapezoidal fuzzy numbers are retained as the basic representation of uncertain transportation costs, while a declared ranking rule supplies scalar values for row reduction, column reduction and route selection. SZM gives priority to a zero that is unique in an active row or column. ZCM instead measures the scarcity of each zero through dynamic row and column zero counts and prefers the structurally most constrained zero. Both procedures allocate the maximum feasible amount, update the residual table, reconstruct the fuzzy objective from the original fuzzy costs and verify ranked optimality using the Modified Distribution Method. Two numerical examples are used for comparison. In a 3 by 4 problem, both methods produce the same ranked-optimal plan with cost 381 and fuzzy objective (306, 381, 381, 456; 1). In a 4 by 4 problem, the methods produce different shipment patterns but the same ranked-optimal cost of 381 and fuzzy objective (301, 381, 381, 461; 1), illustrating the presence of alternative optima. The results show that zero-based allocation is transparent and computationally accessible, but its claims should be supported by explicit ranking assumptions, deterministic tie rules and an independent optimality check.

Keywords: Fuzzy transportation problem, generalised trapezoidal fuzzy number, Single Zero Method, Zero Counting Method, zero scarcity, MODI

1. Introduction

The classical transportation problem seeks a shipment pattern that satisfies all source supplies and destination demands at minimum cost. Its linear structure is well established, but the usefulness of an exact cost matrix depends on the quality of the underlying information. Freight rates, fuel exposure, handling charges, contract adjustments and demand-related operating conditions are often described by ranges or expert assessments rather than by stable point values. Fuzzy set theory provides a convenient way to preserve this imprecision within the optimisation model instead of forcing each uncertain coefficient into one crisp number at the outset (Zadeh, 1965; Bellman & Zadeh, 1970) [1, 13].

Fuzzy transportation research has developed several ways of combining uncertainty representation with classical network optimisation. Some approaches use ranking functions to compare fuzzy coefficients, whereas others use extension-principle calculations or modified allocation procedures (Chanas *et al*., 1984; Chanas & Kuchta, 1996; Liu & Kao, 2004) [2, 3, 10]. The practical difficulty is that a constructive transportation algorithm needs explicit ordering at several stages. A row minimum, a column minimum and a preferred allocation cell cannot be chosen unless the fuzzy costs are compared by a declared rule. If that rule is left implicit, the resulting solution is difficult to reproduce and the meaning of optimality becomes unclear.

This paper develops a common mathematical framework for two zero-based direct procedures. The first is the Single Zero Method, which gives priority to rows or columns containing a unique zero after opportunity-cost reduction. The second is the Zero Counting Method, which quantifies local scarcity by counting the zeros available in each active row and column. The two methods are deliberately compared under the same ranking, reduction, allocation and verification structure. The aim is not to replace exact optimisation, but to examine whether a transparent zero pattern can construct a strong basis while retaining the original fuzzy cost information for final interpretation.

2. Fuzzy Transportation Formulation

Let there be m sources and n destinations. The shipment x_ij from source i to destination j is non-negative, and the corresponding unit cost is the fuzzy number c-tilde_ij. Source availability and destination requirement may also be fuzzy in a broader model, but the examples considered here use representative rim quantities so that an implementable shipment matrix can be formed. The fuzzy objective is written as follows.

$$\min \tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\, x_{ij} \tag{1}$$

The constraints retain the usual transportation equalities: shipments leaving each source equal its available quantity and shipments entering each destination equal its requirement. A balanced problem has equal total supply and demand. When the totals differ, a dummy destination or dummy source may be introduced, with the economic meaning of the dummy route stated explicitly rather than treated as a purely mechanical device.

Generalised trapezoidal fuzzy numbers are used for route costs. A number A-tilde = (a1, a2, a3, a4; w) contains lower and upper support values, a core interval and a maximum membership height w. The representation can therefore distinguish the most strongly supported cost band from less compatible values near the support limits. For route comparison, the study adopts the height-weighted arithmetic-mean ranking used in the generalised trapezoidal SZM literature (Samuel & Raja, 2018) [12].

$$R(\tilde{A}) = \frac{w}{4}(a_1 + a_2 + a_3 + a_4),\ \tilde{A} = (a_1, a_2, a_3, a_4; w) \tag{2}$$

The ranking value is a comparison index rather than a replacement for the fuzzy number. Original fuzzy coefficients remain attached to their routes throughout the calculation. This separation is important because two fuzzy costs can have similar ranked magnitudes but different spreads. Allocation is performed on comparable representatives, while the final objective is reconstructed from the original fuzzy costs.

3. Common Zero-Based Reduction

Both SZM and ZCM begin with the same opportunity-cost table. Each ranked row is reduced by its minimum and each resulting column is then reduced by its minimum. The operation guarantees at least one zero in every active row and column.

$$c'_{ij} = R(\tilde{c}_{ij}) - \min_k R(\tilde{c}_{ik}),\ c''_{ij} = c'_{ij} - \min_k c'_{kj} \tag{3}$$

A zero identifies a route that is locally competitive after source and destination cost levels have been removed. The zero pattern is not itself an optimality certificate. In particular, a reduced matrix may contain several zeros whose local attractiveness creates different feasible allocation sequences. The selection rule therefore determines how the direct basis is built. If the current zero network cannot support the residual source and destination quantities, the reduced table may be revised by a minimum uncovered value in the usual assignment-style manner until adequate zero support is obtained.

Once a zero cell is selected, the largest feasible shipment is assigned. This choice guarantees that at least one active row or column is exhausted at every positive allocation and therefore makes the direct construction finite.

$$x^{*}_{ij} = \min\{a_i^{(r)}, b_j^{(r)}\} \tag{4}$$

4. Single Zero Method and Zero Counting Method

4.1 Single Zero Method

SZM interprets uniqueness as urgency. If an active row has only one zero, that route is its only currently zero-opportunity option. The same argument applies to a destination column with a single zero. The procedure therefore gives first priority to zero cells that are unique in either dimension. When several single-zero candidates exist, the larger feasible allocation is preferred; a remaining tie is resolved by the lower original ranked cost and then by fixed row-column order. This hierarchy removes ambiguity from hand calculations and supports direct software implementation.

The method is especially intuitive in sparse zero tables because a unique zero represents a locally constrained source or destination. Its limitation is also clear: when many rows and columns contain multiple zeros, uniqueness alone offers little discrimination. This motivates a continuous scarcity measure rather than a binary single-zero indicator.

4.2 Zero Counting Method

ZCM recomputes the number of zero entries in each active row and column after every allocation. Let r_i be the active row-zero count and q_j the active column-zero counts.

$$r_i = \sum_{j \in J_r} I\,[c''_{ij} = 0], \qquad q_j = \sum_{i \in I_r} I\,[c''_{ij} = 0] \tag{5}$$

For every active zero, the scarcity score is defined by the reciprocal of these counts. A zero in a row or column with few alternatives receives a larger score than a zero surrounded by many alternatives.

$$\sigma_{ij} = \frac{1}{r_i} + \frac{1}{q_j}, \qquad (i,j) \in Z_r \tag{6}$$

The cell with the largest scarcity score is selected. Ties are resolved by the largest feasible shipment, the lower original ranked cost and fixed position. The score has a useful graph interpretation. If sources and destinations are vertices in a bipartite graph and every zero is an edge, r_i and q_j are vertex degrees. ZCM favours edges incident to low-degree vertices. SZM can therefore be viewed as the limiting case in which degree one receives absolute priority, whereas ZCM continues to distinguish cells when all degrees exceed one.

Table 1: Operational comparison of SZM and ZCM

Element	Single Zero Method	Zero Counting Method
Primary signal	A zero unique in an active row or column	Dynamic scarcity of each zero
Structural measure	Binary uniqueness	Row and column zero counts
Priority rule	Single zero, then largest feasible allocation	Largest 1/r_i + 1/q_j, then largest feasible allocation
Best suited to	Sparse zero patterns	Dense or highly tied zero patterns
Verification	MODI or linear programming	MODI or linear programming

5. Optimality Verification

A direct transportation procedure can produce a feasible plan without proving that the plan is globally optimal. The present framework therefore separates construction from certification. For every basic cell, row and column potentials satisfy u_i + v_j = R(c-tilde_ij). Reduced costs are then calculated for non-basic cells. In a minimisation problem, non-negativity of every non-basic reduced cost is the MODI optimality condition.

$$\Delta_{ij} = R(\tilde{c}_{ij}) - (u_i + v_j), \qquad \Delta_{ij} \geq 0 \quad \forall (i,j) \notin \mathcal{B} \tag{7}$$

If a negative reduced cost is found, the corresponding route enters the basis and a closed plus-minus loop improves the solution. This check is important for methodological credibility because it prevents the termination rule of a heuristic from being mistaken for an optimality theorem. It also identifies alternative optima: a non-basic route with zero reduced cost may enter through a feasible loop without changing the ranked objective.

6. Numerical Evaluation

6.1 Example 1: 3 by 4 balanced problem

The first numerical problem has supplies of 20, 30 and 25 units and destination demands of 18, 20, 15 and 22 units. Total supply and demand are both 75. Each ranked route cost c is represented by the symmetric generalised trapezoidal fuzzy number (c-1, c, c, c+1; 1), so the ranking in Equation (2) returns exactly c. This construction keeps the allocation arithmetic visible while retaining an uncertain final cost.

Table 2: Ranked transportation cost matrix for Example 1

Source	D1	D2	D3	D4	Supply
S1	14	8	5	12	20
S2	5	9	7	9	30
S3	6	8	12	2	25
Demand	18	20	15	22	75

After row and column reduction, SZM and ZCM follow the same effective priority sequence and produce the same feasible plan: 5 units from S1 to D2, 15 from S1 to D3, 18 from S2 to D1, 12 from S2 to D2, 3 from S3 to D2 and 22 from S3 to D4. The ranked transportation cost is 381. MODI gives no negative reduced cost, so this plan is optimal for the ranked problem.

The fuzzy objective is not obtained from the reduced table. It is reconstructed by multiplying each original fuzzy route cost by its shipment and adding all route contributions.

$$\tilde{Z}(X) = \oplus_{i=1}^{m} \oplus_{j=1}^{n} x_{ij} \tilde{c}_{ij} \tag{8}$$

Because 75 units are transported and each selected unit cost has a symmetric support one unit below and above its ranked centre, the resulting fuzzy objective is (306, 381, 381, 456; 1). Thus 381 is the ranked centre of an uncertainty band rather than the only information reported by the model.

6.2 Example 2: alternative optimal plans

The second problem uses four sources with supplies 18, 22, 25 and 15 and four destinations with demands 20, 15, 25 and 20. The total quantity is 80. Its ranked cost matrix is given below.

Table 3: Ranked transportation cost matrix for Example 2

Source	D1	D2	D3	D4	Supply
S1	10	7	7	6	18
S2	5	9	14	4	22
S3	12	3	4	2	25
S4	9	14	6	5	15
Demand	20	15	25	20	80

Here the methods do not end with the same shipment pattern. SZM allocates S1-D2 = 8, S1-D3 = 10, S2-D1 = 20, S2-D4 = 2, S3-D2 = 7, S3-D4 = 18 and S4-D3 = 15. ZCM instead allocates S1-D3 = 18, S2-D1 = 20, S2-D4 = 2, S3-D2 = 15, S3-D4 = 10, S4-D3 = 7 and S4-D4 = 8. Despite the different logistics, both plans have ranked cost 381. MODI verification confirms that both are ranked-optimal, showing that the zero structure can encode a family of alternative optima rather than one unavoidable basis.

Table 4: Comparative numerical results

Criterion	Example 1 SZM	Example 1 ZCM	Example 2 SZM	Example 2 ZCM
Ranked objective	381	381	381	381
Fuzzy objective	(306,381,381,456;1)	(306,381,381,456;1)	(301,381,381,461;1)	(301,381,381,461;1)
Positive allocations	6	6	7	7
Ranked optimality	Verified	Verified	Verified	Verified
Final plan identical?	Yes	Yes	No	No

7. Discussion

The numerical results support three observations. First, the two direct procedures can converge to the same optimum when the reduced zero structure contains sufficiently strong local signals. In Example 1, uniqueness and scarcity identify effectively the same constrained routes. Second, different local rules can generate different members of the optimal set. Example 2 is therefore not evidence that one method failed. It shows that transportation optimality may coexist with distinct route structures, which can be useful when a decision-maker has secondary criteria such as reliability, number of active routes, carbon exposure or upper fuzzy cost.

Third, fuzzy representation and allocation logic should not be conflated. The ranking function is needed to order costs, but the original trapezoidal numbers carry information about imprecision. If the fuzzy coefficients were permanently replaced by ranks, the final result would conceal the support width that motivated fuzzy modelling. Retaining the original coefficients permits alpha-cut analysis, comparison of support widths and sensitivity to alternative ranking weights. Where several plausible rankings lead to the same basis, the recommendation is structurally more stable. Where the basis changes, the model is preference-sensitive and that dependence should be reported.

The methods also have clear limitations. Their zero-selection rules are local. Neither the presence of a single zero nor a high scarcity score evaluates every future allocation sequence. Highly symmetric matrices can produce repeated ties, and a local choice can occasionally require post-optimisation. For that reason, SZM and ZCM are most defensible as transparent constructive procedures or strong initial-basis generators, followed by MODI or an exact network solver. Their computational appeal lies in simple table operations, but the simplicity should not be converted into an unsupported universal optimality claim.

Practical use also depends on how fuzzy costs are elicited. A four-coordinate trapezoid should have an operational interpretation, for example a plausible low cost, a lower core value, an upper core value and a plausible high cost. Membership height should not be treated as a probability. If height represents confidence rather than desirability, the ranking function may need modification because multiplying by a low height can make an uncertain high-cost route appear artificially attractive. Sensitivity analysis is therefore part of responsible fuzzy optimisation rather than an optional embellishment.

7.1 Reproducibility, computational behaviour and implementation

For reproducible implementation, the two procedures can be coded within one common routine. The input layer stores the original fuzzy cost matrix and the source and destination rims. A ranking function produces a separate numerical matrix, which is reduced only for allocation decisions. At every iteration the active source and destination sets are updated, the reduced matrix is recomputed, and the relevant zero statistic is evaluated. In SZM mode the statistic is the existence of a row or column with one zero; in ZCM mode it is the reciprocal scarcity score. Keeping the ranked matrix separate from the fuzzy data avoids a common implementation error in which reduced costs overwrite the original coefficients and make reconstruction of the fuzzy objective impossible.

Computational effort can be assessed through more than the final objective value. Useful indicators are the number of positive allocations, reduction cycles, tie events and MODI pivots needed after direct construction. A method that reaches the optimum with fewer post-optimisation pivots has produced a stronger initial basis, even when another method eventually reaches the same objective. In the two examples considered here, both procedures generated verified optima without a difference in ranked cost, but the second example showed that their route choices can diverge. This suggests that benchmarking should record basis structure as well as cost because two algorithms may be economically equivalent while imposing different operational loads on the network.

The treatment of ties is also part of reproducibility. A paper that states only “select the best zero” leaves an important degree of freedom to the analyst. The present hierarchy uses structural priority first, then maximum feasible shipment, then original ranked cost and finally fixed position. Other tie rules are possible, but they should be declared in advance. This is particularly important in fuzzy problems because an apparently minor tie can change the set of routes used and therefore change the spread of the reconstructed fuzzy objective even when the ranked centre remains unchanged.

For larger instances, SZM or ZCM can be used as initial-basis generators for an exact transportation or linear-programming solver. This hybrid use preserves the interpretability of zero-based reasoning while allowing rigorous optimisation at scale. It also creates a practical route for empirical comparison: the direct objective gap, number of MODI pivots and stability of the selected basis under alternative ranking functions can be measured across a test set. Such benchmarking would provide stronger evidence about when uniqueness or zero scarcity offers a computational advantage.

8. Conclusion

This paper presented a unified comparison of the Single Zero Method and Zero Counting Method for fuzzy transportation problems. Both methods operate on a common ranked opportunity-cost table, allocate the maximum feasible quantity, retain the original fuzzy route coefficients and use MODI for independent optimality verification. SZM prioritises unique zeros, whereas ZCM uses dynamic row and column zero counts to measure scarcity. In the first numerical example both methods produced the same ranked-optimal cost of 381 and the same fuzzy objective. In the second, they produced different but equally optimal shipment plans, demonstrating the usefulness of zero-based procedures for revealing alternative bases. The main methodological implication is that direct fuzzy transportation methods should make four elements explicit: the fuzzy representation, the ranking rule, the cell-selection logic and the optimality certificate. When these elements are documented, zero-based allocation provides a reproducible and interpretable route from uncertain cost data to a verified transportation plan.

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