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Homotopy Theory and Cohomological Structures in Univalent Type Theory

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Abstract

Homotopy Theory has become one of the most powerful fields of the modern mathematics, creating strong links between algebraic topology and category theory, algebraic geometry and computational logic. The goal of Univalent Type Theory (UniTT+hit) has been to provide a constructive and computational framework that is able to precisely and logically capture homotopy-theoretic ideas in recent years. In the present paper, we discuss how to construct homotopy groups, pushouts, suspensions, wedges, pointed types, CW complexes and Eilenberg-Steenrod cohomology in Univalent Type Theory. Special focus is paid to the interpretation of equality as identification as path, which allows to build up higher-dimensional groupoids and homotopy structures. The paper also touches on the application of UniTT+hit to automate homotopy theoretic proofs and to constructively use cohomological structures using computational techniques. New ideas like cellular cohomology, transport, finite dimensional CW complexes, and reduced cohomology theories are discussed algebraic and topologically. As another significant contribution to the theory of homotopy, the equivalence conjecture between cellular cohomology and ordinary reduced cohomology is studied. The use of constructive mathematics and computational reasoning will assist in developing the foundations of homotopy theory, while also moving towards the goal of verified and mechanized mathematics.

Keywords: Homotopy Theory, Univalent Type Theory, Cohomology Theory, CW Complexes, Eilenberg–Steenrod Cohomology, etc.

Introduction

One of the main subjects of algebraic topology that is about continuous transformations between topological spaces and mappings is known as Homotopy Theory. The theory is mainly concerned with properties which are preserved under homotopic equivalence and offers strong algebraic methods for distinguishing spaces by means of homotopy groups and cohomology groups. Homotopy Theory began as a branch of algebraic topology but has branched out over time into a part of category theory, algebraic geometry, higher dimensional algebra, and modern computational logic. Constructive approaches like Intensional Type Theory and Univalent Type Theory have had a major impact on the way that homotopical structures are represented and mathematically verified. The development of Homotopy Type Theory created a deep connection between equality and connecting paths in geometry. The identification-as-path interpretation added a way of relating the process of logical proof to higher-dimensional topology, by interpreting equality proofs as paths in spaces. Further, this interpretation allowed to construct ∞-groupoids, homotopy groups, pushouts, suspensions and cohomological structures directly in type-theoretic systems. As a result, the equality no longer is only a syntactic relation but one with geometric and computational meaning.

Higher inductive types combined with Univalent Type Theory (UniTT+) is a constructive framework for automation of homotopy-theoretic proofs and formalization of topological structure. The framework provides a way to represent spaces, spheres, pushouts, wedges, and CW complexes in a meaningful way in a computational sense. Utilizing path induction and higher dimensional identifications, UniTT+hit makes possible the rigorous development of ordinary homotopy theory and cohomology theory. These constructions are also instrumental to the computational interpretation of such constriction, which in turn helps develop machine-verified mathematics and proof assistants like Agda and Lean. Among the most significant ideas to be considered in this context, the study of cohomology groups and homotopy groups is one. Homotopy groups and cohomology groups offer algebraic tools for distinguishing spaces by higher dimensional loops and cycles, and mappings and cycles satisfying algebraic structures, respectively. While higher homotopy groups are frequently easier to calculate, cohomology groups are also important tools in classical and modern topology. The constructive interpretation of the concepts of CW complexes and cellular cohomology in UniTT+hit gains additional support from the development of cellular cohomology and CW complexes inductively, by cells of increasing dimension, describing topological spaces. In the present paper, the role of UniTT+hit, in formalizing Homotopy Theory and cohomological structures, is discussed. Special focus is given to homotopy groups, pushout, wedges, pointed types, suspensions, CW complexes, cellular cohomology and Eilenberg–Steenrod cohomology. The paper also contains a discussion of the equivalence conjecture between the cellular cohomology and the ordinary reduced cohomology theories in finite dimensional CW complexes. The study shows how important constructive type-theoretic approaches are in current modern topology and also serves as an introduction to the goal of "computationally verified mathematics.

Literature Review

Frédéric Déglise (2025) [4] The notes are a transcription of a talk that Bourbaki gave on the motivic homotopy theory of Morel-Voevodsky over a field. Calculations of stable and unstable motivic homotopy sheaves are the main focus of these notes. This paper also describes the applications of Isaksen-Wang-Xu to the problem of determining stable stems from the motivic Adams spectral sequence. Theorizing synthetic homotopy was made possible in part by these uses. You will find descriptions of both of these uses.

Adrian Clough (2024) [14] We show in this research that the $\mathscr{J}$-category of global spaces is similar to the homotopy localization of the $\mathscr{J}$-category of sheaves on the site of topology of separated differentiable stacks. This is in accordance with the philosophy presented by Gepner-Henriques. Additionally, we provide additional evidence that this ∞-category of sheaves is a cohesive ∞-topos, and that it includes the singular-cohesive ∞-topos of Sati-Schreiber in a complete and true manner.

Charlotte Aten (2024) In 2017, Walter Taylor presented a presentation that established the presence of two-dimensional simplicial complexes that accept the structure of topological modular lattice but do not admit the structure of topological distributive lattice. This demonstration was successful in demonstrating the existence of these complexes. An answer that is favorable is provided by us in response to his query about the existence of n-dimensional simplicial complexes that exhibit the same property. For the purpose of doing this, we provide, for any n that is larger than or equal to two, an infinite family of compact simplicial complexes that accept the structure of topological modular lattice but do not admit the structure of topological distributive lattice.

Mahima Ranjan Adhikari (2023) [2] A significant contribution made by H. In his landmark thesis titled "Analysis Situs," which was published in Paris in 1895, Poincaré (1854–1912) set the framework for homotopy theory, which is especially treated in this chapter. Poincaré contributed contributing to the advancement of homotopy theory. A. H. In the article that he published in 1935 discussed these classifications. The study of continuous maps and topological spaces that are characterized by features that remain invariant under homotopic mappings is the focus of the domain of homotopy theory. There are additional topological spaces included in this category.

T Rupavani (2023) [5] The scientific investigation of certain situations in which two maps are homotopies is referred to as the theory of homotopy. Although it was once a subject of study within the field of algebraic topology, it is currently being investigated independently. In addition to its use in algebraic topology, the notion has also been utilized in the analysis of category theory and algebraic geometry. In the course of this investigation, we will make use of Homotopy Theory as well as homological invariants of topological and geometrical objects. Taking into consideration group theory, khovanov homology, and homotopy theory for digraphs, the emphasis of this essay was on the links that exist between homology and homotopy groups of spaces.

Homotopy Theory in Univalent Type Theory

UniTT+hit, provides a practical basis for automating homotopy theory is backed up by this body of evidence, which gives overwhelming support for my claim. The work that we did on the Blakers-Massey theorem resulted in the introduction of new methodologies, which even encouraged another mathematical research.

Table 1: The identification-as-path interpretation

Type Theory	**Homotopy Theory**
A: type	space
$a: A$ (element)	point
$f: A \to B$ (function)	continuous mapping
$A \to B$ (arrow type)	function space
$A \to \mathcal{U}$ (family of types)	fibration
$B(a)$ (instance of a family)	fiber
$b(x): B(x)$ (conditional element)	section
$\sum_{x:A} B(x)$ (sum type)	total space
$\prod_{x:A} B(x)$ (function type)	space of sections
$a =_A b$ (identification)	path

The new framework takes into account the possibility of automating proofs that already exist as well as the creation of proofs that are completely new. In the field of homotopy theory, two fundamental ideas are known as homotopy groups and cohomology groups. These algebraic techniques make it possible for us to differentiate spaces that are homotopically distinct from one another. Cohomology groups are concerned with mappings from cycles in a space satisfying the requirements of an abelian group, and homotopy groups are concerned with techniques that fold the sphere that is dimensional into a space. Either the nth homotopy or the cohomology group is the group that represents the nth dimension in both of these instances. Since cohomology groups are often simpler to compute for pertinent spaces, they are attractive. May be partially ascribed to the infamous difficulty of calculating higher homotopy groups. This contrasts with the initial homotopy groups, usually referred to as basic groups, which are generally understood. We will see that covering spaces are tightly linked to fundamental groups and may be succinctly expressed using UniTT+hit. By taking use of smaller spaces, the Seifert-van Kampen theorem calculates the basic group of a bigger space using a divide-and-conquer approach. Higher homotopy groups may be calculated using the Blakers-Massey theorem, which is one of the few methods currently accessible. The following three findings are helpful tools for homotopy groups wherever they are applied to classical theory. In UniTT+hit, cohomology theory is still a relatively undiscovered field; nonetheless, the most recent conclusion follows a pattern that emerged more recently by expanding this collaborative program to include it. The objective of linking the multiple UniTT+hit cohomology models is advanced by the work that we have done in collaboration with Ulrik Buchholtz.

Primary Groupings of Entity

For ∞-groupoids, iterative identification is the fundamental element. Therefore, commonly referred to as basic groupoids, ordinary groupoids are produced by the 0-truncation of identification. It. Focusing on shorter identification at a particular aspect causes them to fall into simple groups. $\mathbb{Z}$ is the basic group of Π1. In formal terms, regardless of type A, the groupoid of A, denoted as λ (a, b: A), is created by the arrows of the groupoid with concatenation as composition and reflexivity as the unit, where $\|a = A\ b\|0$. The basic group of type A at a, denoted as $\pi1$ (A, a), is the set $\|a = a\|0$ under the same administration, given a distinct member a: A. Concatenation, reflexivity, inversion, and ap$f$ operators for 0-truncated identifications will be reused for clarity's sake; however, for untruncated identifications, these operators are distinct. When passing an identity across a family of sets, the truncation level is indicated by the subscript 0. which is expressed as transport0 [x. $B(x)$] (p; a).

Pushouts and Friends: An example of Two kinds, A and B, are joined by a homotopy pushout with identifications on both sides; a different type C indexes the additional identifications, which are written glue, and functions. $f : C \to A$ and $g : C \to B$ signifying the glue's termination places (c). The expression for the pushout in type theory is $A \sqcup_{C;\, f;\, g} B$, where A and B are represented as left and right, respectively; refer to the picture for reference. The formal rules of it are 4.7. The f and g in $A \sqcup_{C;\, f;\, g} B$ might be left out if it is obvious from the context. A pushout, from a category theory perspective, is a colimit of the span.

$$\begin{tikzcd} A & C \arrow[l, "f"'] \arrow[r, "g"] & B \end{tikzcd}$$

This is a pushout square (commuting square) starting in the bottom right corner.

$$\begin{tikzcd} C \arrow[r, "g"] \arrow[d, "f"'] & B \arrow[d, "\mathtt{right}"] \\ A \arrow[r, "\mathtt{left}"'] & A \sqcup_{C;f;g} B \end{tikzcd}$$

The pushout construction can be employed to generate a variety of objects of homotopy-theoretic significance, including the set quotients, the circle, the cofibers, the suspensions, and the expressive cellular complexes. The cofibers and suspensions are as follows:

Categories of suspensions. Initially, the type A seems to be suspended from two poles, north and south, below. Their formal definition follows:

$$\begin{aligned} \mathtt{susp}(A) &:\equiv \mathbb{1} \sqcup_A \mathbb{1} \\ \mathtt{north} &:\equiv \mathtt{left}(\mathtt{unit}) : \mathtt{susp}(A) \\ \mathtt{south} &:\equiv \mathtt{right}(\mathtt{unit}) : \mathtt{susp}(A) \\ \mathtt{merid}(a{:}A) &:\equiv \mathtt{glue}(a) : \mathtt{north} = \mathtt{south} \end{aligned}$$

$$\begin{tikzcd} A \arrow[r] \arrow[d] & \mathbb{1} \arrow[d, "\lambda\_.\mathtt{south}"] \\ \mathbb{1} \arrow[r, "\lambda\_.\mathtt{north}"'] & \mathtt{susp}(A) \end{tikzcd}$$

Wedges and Pointed Types of Tools

A pointed type is one that has an identifiable feature, or more specifically, two types that are pointed. $\sum_{A:\mathrm{U}} A$. We provide unique projections for carriers and points (pt) to enhance the presentation of the findings.

$$\mathtt{carrier} : \sum_{A:\mathcal{U}} A \to \mathcal{U} \qquad \mathtt{pt} : \prod_{X:\sum_{A:\mathcal{U}} A} \to \mathtt{carrier}(X)$$

$$\mathtt{carrier} :\equiv \mathtt{fst} \qquad \mathtt{pt} :\equiv \mathtt{snd}$$

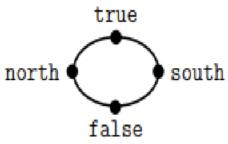
Assume Y and Y are two kinds with points. The functions that maintain the distinguishing element are collected by pointed arrow types from Y to Y, which are expressed as $Y \cdot\!\!\to Y$.

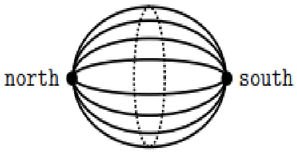
$$X \cdot\!\!\to Y :\equiv \sum_{f:\mathtt{carrier}(X)\to\mathtt{carrier}(Y)} f(\mathrm{pt}(X)) = \mathrm{pt}(Y).$$

It should be noted that the constant function λ may be used to make the type $X \cdot\!\!\to Y$ so that it is pointed. However, based on my experience with Agda, I can say that the use of pointed arrow types will simply result in more difficult proofs, especially in situations where there is no implicit coercion. A lot of the types that have been discussed up to this point may be recognized by the characteristic that sets them apart from one another. When it is provided, the unique element A is the source of the pushout's distinctive feature, like $A \sqsubset \mathbf{O}\ B$. Similarly, types defined by pushouts are also supplied. The symbols frequently used for their pointed counterparts will be repurposed for these types. For pointed arrows (Z) and pointed types (X), the particular versions that are taken into account are cofiber (f) and susp (X).

Booleans as the First Sphere

The principles in Figure may be used to represent the standard Boolean type $\mathbb{2}$. In addition to being equal to $\mathbb{S}1$, the Boolean type suspension, susp ($\mathbb{0}$), yields the two-dimensional sphere when $\mathbb{S}1$ is suspended with the circle as the equator.





For convenience, we may refer to $\mathbb{2}$ as $\mathbb{S}0$ because $\mathbb{S}1$ is the suspension of $\mathbb{2}$, and the idea of spheres as iterated suspensions is derived from This results in the $(n + 1)$-dimensional sphere from suspending the n-dimensional sphere.

$$\mathbb{S}^n :\equiv \mathtt{elim}_{\mathbb{N}}[\_.\mathcal{U}](\mathbb{2};\_s.\mathtt{susp}(s);n).$$

The two definitions of □1! are equal. Based on our Agda mechanization, the picture-inspiring rules are established by the circle as the suspension of the Boolean type. 4.10, which includes assessing equality, may be efficiently implemented using several rules. Therefore, in my thesis, I purposefully use the same sign for both $\mathbb{S}1$'s.

Study of Ordinary Eilenberg–Steenrod Cohomology

Moving on from homotopy groups, we now talk about cohomology groups, which are interested in the type-cycle structure of functions. The study of cohomology groups, also called cohomology theory, is often thought of as a part of homotopy theory because it is based on the idea of has significant ties to accurate homotopy theory and is homotopy equivalent. It is puzzling that these types of groups may be defined in both combinatorial and logical ways, given that classical theory asserts that they are essentially equivalent. We want to get Agda and UniTT+hit to be the same. This part is all about CW complexes, a group of types that clearly show how they are put together from structures with lower dimensions to ones with higher dimensions. The next part will talk about cellular cohomology theory, which is a combinatorial theory of cohomology that only works for CW complexes. After explaining what ordinary Eilenberg-Steenrod cohomology theory is, which is a basic idea for cohomology theory, we will then present our equality hypothesis. Next, a basic idea that supports the theory is explained and shown.

Table 2: Abuse of notation

Notation	Types	Pointed types	Groups
(A ⟶B)	arrows	pointed arrows (only in diagrams)	group homomorphisms
$\prod_{a:A} B$	functions	(not reused)	direct products
A≃ B	equivalence	point-preserving equivalence	group isomorphism
$\mathbb{0}$	the empty type	(not reused)	the trivial group
1	the unit type	the unit type	(not reused)
A ×B	binary products	(not reused)	binary direct products
susp(A)	suspensions	pointed suspensions	(not reused)
cofiber(f)	cofibers	pointed cofibers	(not reused)
(A / B	set quotients	pointed cofibers of inclusions	group quotients

As shown in Table 2, I use the arrow (→), functions (∏), equality (≃), the unit ($\mathbb{1}$), binary products (×), and other symbols for groups and types with points again and again in this part. The pointedness of functions is significant in the subsequent discussion of degrees, thus except in diagrams of pointed types, pointed arrows are still shown as $a \cdot\!\!\to Y$.

CW complexes

By connecting cells in a certain order-starting with zeroth-dimensional points, lines in the first dimension, faces in the second dimension, etc.-an inductively defined CW complex may be built. The description is made up of A_n, that is the set of cells with dimension n and functions An that are the links between the cells. Ulrik Buchholtz's contributions to Lean provide the groundwork for the following recursive definition.

The border of a cell $\alpha : An+1$ at dimension $n + 1$ is defined by a function from Xn, where an is the construction up to that dimension. $\mathbb{S}^{n}$ to the type X^n; the type $Xn+1$ when all cells have been attached at a given dimension, $n + 1$ to X_n. More formally, α_{n+1} is a function from $A_{n+1} \times \mathbb{S}^n$ to X_n outlining the boundaries of every single cell. Deductively, the first kind X_0 is the set A_0, and the type X_{n+1} is characterized as the pushout $X_n \sqcup A_{n+1} \times \mathbb{S}^n; \alpha n+1; \mathrm{fst}\ An+1$:

$$\begin{tikzcd}
A_{n+1} \times \mathbb{S}^n \arrow[r, "\mathtt{fst}"] \arrow[d, "\alpha_{n+1}"'] & A_{n+1} \arrow[d] \\
X_n \arrow[r] & X_{n+1}
\end{tikzcd}$$

First approximation: the construction process finishes at some finite dimension as only finite-dimensional CW complexes are taken into account. The iterated pushout that begins with the type $X_0 :\equiv A_0$ and ends at some dimension is graphically represented as a (finite) CW complex.

$$\begin{tikzcd}
 & A_{n+1} \times \mathbb{S}^n \arrow[r, "\mathtt{fst}"] \arrow[d, "\alpha_{n+1}"] & A_{n+1} \arrow[dr] & A_{n+2} \times \mathbb{S}^{n+1} \arrow[r, "\mathtt{fst}"] \arrow[d, "\alpha_{n+2}"] & A_{n+2} \arrow[dr] & & \\
\cdots \arrow[r] & X_n \arrow[rr] & & X_{n+1} \arrow[rr] & & X_{n+2} \arrow[r] & \cdots
\end{tikzcd}$$

Notably, in order for A_0 and all subsequent pushouts to be pointed, A_0 itself must be pointed for a pointed CW complex.

Cellular Cohomology

Cohomology theory is concerned with functions that are obtained from cycles, and a good way to start is with homology theory, which considers the cycles in isolation. Proper cycles may be administered algebraic structures once a type is well characterized, whether it simplicial or cellular. Our focus will be on the situation when a CW complex X and its cellular description are given. In the concepts of homology and cohomology, a one-dimensional cycle is a collection of oriented lines that may be connected linearly to form a set of cycles in graph theory that can overlap. The linear combination of each coefficient in a homology-theoretic cycle reflects the frequency of a line within these graph-theoretic cycles, with negative values attributed to traversals in the opposite direction. Two traversals in opposite directions cancel each other out, therefore the order of the lines doesn't matter when determining a linear combination; what does important is the direction of the lines. One may further define operations on these homology-theoretic cycles, including addition, subtraction, and negation; in particular, the same graph-theoretic cycles in the other direction are formed by negating a homology-theoretic cycle. From here on, cycles will be used to describe homology-theoretic cycles. Assume $\sigma 1$. The function may be used to transform a line from a to b into the linear combination $a - b$, which represents the line's orientation boundary. When it comes to graph theory, every vertex in a cycle has the same number of "ins" and "outs," meaning it adds zero to the linear combination net. If a collection of lines is a collection of graph-theoretic cycles, then their total is zero.

Eilenberg–Steenrod Cohomology

In contrast to the clear building we just talked about, cohomology has a logical basis. The well-known abstract basis for cohomology theories, the Eilenberg–Steenrod axioms, have been added to UniTT+hit in the works of authors such as Michael Shulman, Dan Licata, Eric Finster, Peter LeFanu Lumsdaine, and Guillaume Brunerie. A contravariant functor is the best way to describe a reduced cohomology theory in UniTT+hit that maps pointed types to a set of abelian groups that follow certain rules. We use $\hbar\ n$ (X) to show the nth group in the list for a pointed type X. Before talking about these axioms, it's important to explain what it means to meet the set-level axiom of choice, which says that II It is possible to combine quantifiers with 0-truncation. This will be used in one of the cohomology principles that will be shown next.

This is an explanation of 4.5.2 (axiom of choice at the set level). The set-level axiom of choice says that a type A is choice-free if the unchoosing function is an equality relation for every set of types B that A points to.

$$\lambda f.\lambda(i{:}I).\mathtt{elim}_{\|-\|}\Big[\_.\|W(i)\|_0;\ \_.\|W(i)\|_0\text{-}\mathtt{level}\Big]\big(f'.|f'(i)|_0; f\big) : \Big\|\prod_{i:I} W(i)\Big\|_0 \to \prod_{i:I}\|W(i)\|_0$$

The axiom of choice and what it means in cohomology theory are talked about in UniTT+hit. It is not necessary to use the axiom of choice in order to clarify the Eilenberg-Steenrod axioms. Nevertheless, these axioms within UniTT+hit would be difficult to uphold for directed arrows with codomains such as Eilenberg-Mac Lane spaces, a prime example of a cohomology theory. This is the foundation of UniTT+hit:

Holding back. There is an isomorphism between $hn+1(\mathrm{susp}(X))$ and $hn\ (X)$. and it makes sense for X to have that isomorphism.

Equivalence Conjecture and Partial Results

Theorem 1: Given an integer, a pointed finite-dimensional CW complex a, and any standard reduced cohomology theory h, $n, hn\,(X)$ is isomorphic to $H^n\,(X; h^0\,(\mathbb{2}))$. This essentially asserts that, in CW complexes, two concepts of cohomology are equivalent. The importance lies in the linkage of a specific construction to a somewhat abstract framework across a diverse array of kinds. To substantiate this prediction, we may need further assumptions to regain some efficacy of classical reasoning, such is the need for observable cell equality or the use of the set-level rule of choice by groups of cells. We will divide this supposition into two components. For $n \geq m$, we denote $Xn\,/Xm$ as the cofiber of the inclusion from Xm to Xn.

In the diagram given earlier, one of the points-more precisely, the differentiating element-acts as the center. In light of this, we must include one set of $h^0\,(\mathbb{2})$ to the left side to achieve financial equilibrium. Additionally, a separator is required to locate the center of the pointed variety. The remainder of the isomorphism $k0$ remains unchanged in all other dimensions. The major unknown at this time is whether or not the two sequences' group morphisms are comparable:

Conclusion

The present work shows how to constructively and meaningfully formalize Homotopy Theory and cohomological structures with Univalent Type Theory. All of these types of together give a tight relationship between topology and type theory. The interpretation via identification as path further reinforces the homotopical sense of equality and higher dimensional structures. The discussion on CW complexes and cellular cohomology demonstrates an example of how higher dimensional topological objects can be inductively presented in UniTT+hit. Likewise, in the type-theoretic formulation of ordinary Eilenberg-Steenrod cohomology, it is possible to combine abstract algebraic entities and constructive proof systems. The equivalence conjecture between cellular cohomology and ordinary reduced cohomology points to an important research area in modern homotopy theory. In conclusion, UniTT+hit proves itself to be a useful tool for automated homotopy-theoretic reasoning, for building verified proofs and for developing the computational foundations of contemporary topology. The theory is used to support classical homotopical constructions, and is also important towards the development of formally verified mathematics in constructive and computational settings.

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