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Solution of Fuzzy Matrix Game Problem by Method of Oddments using Octadecagonal Fuzzy Numbers

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Abstract

In this paper we have considered fuzzy game matrix. The inexact values in the fuzzy game matrix are Octadecagonal fuzzy numbers. We convert the fuzzy valued game problem to a crisp valued game problem by using ranking which we have tried to solve using method of oddments.

Keywords: Fuzzy sets, Octadecagonal Fuzzy Numbers (ODFN), Ranking of Fuzzy Numbers, Fuzzy Game Problem

Introduction

The theory of games started in the 20th century. But the mathematical treatment of games took fire in 1944 when Neumann, J.V. and Morgenstern, O. [15] published their renowned article on "Theory of games and economic behavior". The Neumann's approach utilizes the mini-max principle which involves the elementary idea of minimization of maximum loss.

All the parameters in the fuzzy game problem are fuzzy numbers. The fuzzy numbers can be triangular, trapezoidal, hexagonal, octagonal, hendecagonal, octadecagonal etc. The ranking method using α-cuts, for ranking of Fuzzy numbers was proposed by Basirzadeh [11] in which he has ranked triangular and trapezoidal fuzzy numbers. For Octadecagonal fuzzy numbers, the arithmetic operations, alpha cut, and ranking technique are introduced by Barya V, Sharma A and Badal D [1]. By using this ranking, the Fuzzy Game problem is converted to a crisp value problem, which can be solved using the method of oddments.

Preliminaries

The aim of this section is to throw some light on some notations, notions and results which are used further.

Fuzzy Set: Let X be a non-empty set. A fuzzy set "A" in X is characterized by its membership function A: $\longrightarrow$ X [0, 1] and A(x) is interpreted as the degree of membership of element x in fuzzy A for each x Ɛ X. Complete non-membership is represented by the value zero; complete participation is represented by the value one and intermediate degrees of membership are represented by values in between. The membership function of fuzzy set A is also known as the mapping A.

Fuzzy Number

A fuzzy number "A" is a convex normalized fuzzy set on the real line R, such that:

a. There exists at least one $x_{\circ}$ Ɛ R with $\mu_A(x) = 1$

b. $\mu_A(x)$ is piecewise continuous.

Octadecagonal Fuzzy Number

A Octadecagoanal fuzzy number is denoted as ĀODFN = (a1, a2, a3, a4, a5, a6, a7, a8, a9, a10, a11, a12, a13, a14, a15, a16, a17, a18) and its membership function is given by:

$$\tilde{A}_{ODFN} = \begin{cases} 0 & ;\text{ for } x < a_1 \\ \frac{1}{8}\left(\frac{x-a_1}{a_2-a_1}\right) & ;\text{ for } a_1 \le x \le a_2 \\ \frac{1}{8}+\frac{1}{8}\left(\frac{x-a_2}{a_3-a_2}\right) & ;\text{ for } a_2 \le x \le a_3 \\ \frac{2}{8}+\frac{1}{8}\left(\frac{x-a_3}{a_4-a_3}\right) & ;\text{ for } a_3 \le x \le a_4 \\ \frac{3}{8}+\frac{1}{8}\left(\frac{x-a_4}{a_5-a_4}\right) & ;\text{ for } a_4 \le x \le a_5 \\ \frac{4}{8}+\frac{1}{8}\left(\frac{x-a_5}{a_6-a_5}\right) & ;\text{ for } a_5 \le x \le a_6 \\ \frac{5}{8}+\frac{1}{8}\left(\frac{x-a_6}{a_7-a_6}\right) & ;\text{ for } a_6 \le x \le a_7 \\ \frac{6}{8}+\frac{1}{8}\left(\frac{x-a_7}{a_8-a_7}\right) & ;\text{ for } a_7 \le x \le a_8 \\ \frac{7}{8}+\frac{1}{8}\left(\frac{x-a_8}{a_9-a_8}\right) & ;\text{ for } a_8 \le x \le a_9 \\ 1 & ;\text{ for } a_9 \le x \le a_{10} \\ 1-\frac{1}{8}\left(\frac{x-a_{10}}{a_{11}-a_{10}}\right) & ;\text{ for } a_{10} \le x \le a_{11} \\ \frac{7}{8}-\frac{1}{8}\left(\frac{x-a_{11}}{a_{12}-a_{11}}\right) & ;\text{ for } a_{11} \le x \le a_{12} \\ \frac{6}{8}-\frac{1}{8}\left(\frac{x-a_{12}}{a_{13}-a_{12}}\right) & ;\text{ for } a_{12} \le x \le a_{13} \\ \frac{5}{8}-\frac{1}{8}\left(\frac{x-a_{13}}{a_{14}-a_{13}}\right) & ;\text{ for } a_{13} \le x \le a_{14} \\ \frac{4}{8}-\frac{1}{8}\left(\frac{x-a_{14}}{a_{15}-a_{14}}\right) & ;\text{ for } a_{14} \le x \le a_{15} \\ \frac{3}{8}-\frac{1}{8}\left(\frac{x-a_{15}}{a_{16}-a_{15}}\right) & ;\text{ for } a_{15} \le x \le a_{16} \\ \frac{2}{8}-\frac{1}{8}\left(\frac{x-a_{16}}{a_{17}-a_{16}}\right) & ;\text{ for } a_{16} \le x \le a_{17} \\ \frac{1}{8}\left(\frac{x-a_{17}}{a_{18}-a_{17}}\right) & ;\text{ for } a_{17} \le x \le a_{18} \\ 0 & ;\text{ for } x \ge a_{18} \end{cases}$$

Parametric Form of Octadecagonal Fuzzy Number

The parametric form of Octadecagonal fuzzy number is defined as U = ($P_1(r)$, $Q_1(s)$, $R_1(t)$, $S_1(u)$, $T_1(v)$, $U_1(w)$, $V_1(x)$, $W_1(y)$, $P_2(r)$, $Q_2(s)$, $R_2(t)$, $S_2(u)$, $T_2(v)$, $U_2(w)$, $V_2(x)$, $W_2(y)$) for r ϵ [0,1/8], s ϵ [1/8,2/8], t ϵ [2/8,3/8], u ϵ [3/8,4/8], v ϵ [4/8,5/8], w ϵ [5/8,6/8], x ϵ [6/8,7/8], y ϵ [7/8,1] where,

a. $P_1(r)$, $Q_1(s)$, $R_1(t)$, $S_1(u)$, $T_1(v)$, $U_1(w)$, $V_1(x)$ and $W_1(y)$ are bounded left continuous non-decreasing function over [0,1/8], [1/8,2/8], [2/8,3/8], [3/8,4/8], [4/8,5/8], [5/8,6/8], [6/8,7/8], and [7/8,1].
b. $P_2(r)$, $Q_2(s)$, $R_2(t)$, $S_2(u)$, $T_2(v)$, $U_2(w)$, $V_2(x)$ and $W_2(y)$ are bounded left continuous non-increasing function over [0,1/8], [1/8,2/8], [2/8,3/8], [3/8,4/8], [4/8,5/8], [5/8,6/8], [6/8,7/8], and [7/8,1].

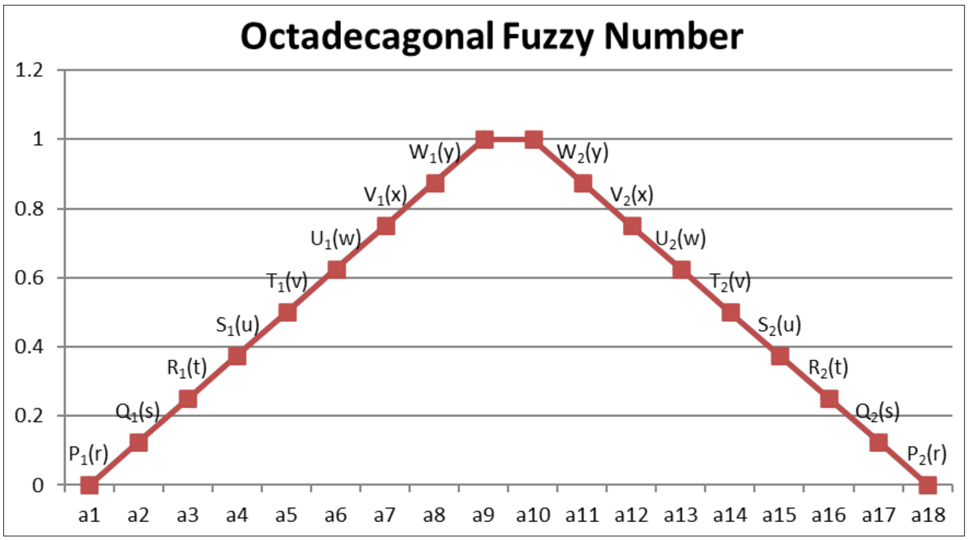

Fig 1: Graphical Representation of Octadecagonal Fuzzy number

Ranking of Octadecagonal Fuzzy Number

The ranking function r:F(R) ⟶ R where F(R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where the natural order exists (Yager 1986), i.e.

a. A > B iff r(A) > r(B)
b. A < B iff r(A) < r(B)
c. A = B iff r(A) = r(B)

Let A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}) and B = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16}, b_{17}, b_{18}) be two Octadecagonal fuzzy numbers then

$$r(A) = \frac{a1 + a2 + a3 + a4 + a5 + a6 + a7 + a8 + a9 + a10 + a11 + a12 + a13 + a14 + a15 + a16 + a17 + a18}{18}$$

And

$$r(B) = \frac{b1 + b2 + b3 + b4 + b5 + b6 + b7 + b8 + b9 + b10 + b11 + b12 + b13 + b14 + b15 + b16 + b17 + b18}{18}$$

Solution of Fuzzy Matrix Game by Method of Oddments

Consider the general (3x3) game matrix, whose elements are Octadecagonal fuzzy numbers.

$$\begin{array}{cc} & \quad\quad\mathbf{B} \\ & \begin{array}{ccc} \mathbf{I} & \mathbf{II} & \mathbf{III} \end{array} \\ \mathbf{A} \begin{array}{c} \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} & \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \end{array}$$

Algorithm

- First we shall convert the given fuzzy game problem into a crisp value problem
- Check for the saddle point in the payoff matrix.
- If there is no saddle point nor it is reducible by dominance, solve using method of oddments.

Step: 1 Subtract each row from the row above i.e. subtract 2nd row from 1st and 3rd row from the 2nd and write the differences in the form of two successive rows beneath the matrix's rows.
Step: 2 Now Subtract each column form the column to its left i.e. subtract 2nd column from 1st and 3rd column from 2nd and write the differences in the form of two successive columns to the right of the matrix.
Step: 3 Now calculate the oddments for the A's 1, 2, 3 strategies and B's I, II, III strategies.
Step: 4 Write these oddments, neglecting the signs.
Step: 5 Now check if the sum of oddments of both the players is same then both the players uses their all pure strategies and hence game is conformable for matrix method. But if the sum of oddments of both the players is different, then both the players do not uses their all pure strategies.
Step: 6 Now divide the oddments by the sum of the oddments to get the optimal strategies of both the players.
Step: 7 Finally calculate the value of game by using the formula given below,

$$V = \frac{\text{Sum of the products of oddments of player A and the corresponding elements of any Row}}{\text{Sum of the oddments}}$$

Or

$$V = \frac{\text{Sum of the products of oddments of player B and the corresponding elements of any Column}}{\text{Sum of the oddments}}$$

Example: Consider the following fuzzy game problem,

$$\text{Player A}\quad \begin{matrix} \\ 1 \\ 2 \\ 3 \end{matrix} \overset{\text{Player B}}{\begin{matrix} \text{I} & \text{II} & \text{III} \\ \left(\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}\right) \end{matrix}}$$

Where, a11 = (-14, -12, -10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20)
a12 = (-16, -14, -12, -10, -8, -6, -4, -2, 2, 3, 4, 5, 6, 7, 8, 9, 20, 26)
a13 = (-16, -14, -12, -10, -8, -6, -4, -2, 2, 3, 4, 5, 6, 7, 8, 9, 20, 26)
a21 = (-16, -14, -12, -10, -8, -6, -4, -2, 2, 3, 4, 5, 6, 7, 8, 9, 20, 26)
a22 = (-16, -14, -12, -10, -8, -6, -4, -2, 2, 3, 4, 5, 6, 7, 8, 9, 20, 26)
a23 = (-16, -14, -12, -10, -8, -6, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24)
a31 = (-16, -14, -12, -10, -8, -6, -4, -2, 2, 3, 4, 5, 6, 7, 8, 9, 20, 26)
a32 = (-9, -4, -3, -2, -1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13)
a33 = (-16, -14, -12, -10, -8, -6, -4, -2, 2, 3, 4, 5, 6, 7, 8, 9, 20, 26)

Solution: First of all we shall use ranking of Octadecagonal fuzzy numbers to convert the given fuzzy game problem to a crisp value problem,

$$R_{ODFN} = \frac{a1+ a2+ a3+ a4+ a5+ a6+ a7+ a8+ a9+ a10+ a11+ a12+ a13+ a14+ a15+ a16+ a17+ a18}{18}$$

a11 = (-14, -12, -10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20)	$R(a11) = \frac{-14+(-12)+(-10)+(-8)+(-6)+(-4)+(-2)+0+2+4+6+8+10+12+14+16+18+20}{18} = \frac{54}{18} = 3$
a12 = (-16, -14, -12, -10, -8, -6, -4, -2, 2, 3, 4, 5, 6, 7, 8, 9, 20, 26)	$R(a12) = \frac{-16+(-14)+(-12)+(-10)+(-8)+(-6)+(-4)+(-2)+2+3+4+5+6+7+8+9+20+26}{18} = \frac{18}{18} = 1$
a13 = (-16, -14, -12, -10, -8, -6, -4, -2, 2, 3, 4, 5, 6, 7, 8, 9, 20, 26)	$R(a13) = \frac{-16+(-14)+(-12)+(-10)+(-8)+(-6)+(-4)+(-2)+2+3+4+5+6+7+8+9+20+26}{18} = \frac{18}{18} = 1$
a21 = (-16, -14, -12, -10, -8, -6, -4, -2, 2, 3, 4, 5, 6, 7, 8, 9, 20, 26)	$R(a21) = \frac{-16+(-14)+(-12)+(-10)+(-8)+(-6)+(-4)+(-2)+2+3+4+5+6+7+8+9+20+26}{18} = \frac{18}{18} = 1$
a22 = (-16, -14, -12, -10, -8, -6, -4, -2, 2, 3, 4, 5, 6, 7, 8, 9, 20, 26)	$R(a22) = \frac{-16+(-14)+(-12)+(-10)+(-8)+(-6)+(-4)+(-2)+2+3+4+5+6+7+8+9+20+26}{18} = \frac{18}{18} = 1$
a23 = (-16, -14, -12, -10, -8, -6, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24)	$R(a23) = \frac{-16+(-14)+(-12)+(-10)+(-8)+(-6)+2+4+6+8+10+12+14+16+18+20+22+24}{18} = \frac{90}{18} = 5$
a31 = (-16, -14, -12, -10, -8, -6, -4, -2, 2, 3, 4, 5, 6, 7, 8, 9, 20, 26)	$R(a31) = \frac{-16+(-14)+(-12)+(-10)+(-8)+(-6)+(-4)+(-2)+2+3+4+5+6+7+8+9+20+26}{18} = \frac{18}{18} = 1$
a32 = (-9, -4, -3, -2, -1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13)	$R(a32) = \frac{-9+(-4)+(-3)+(-2)+(-1)+1+2+3+4+5+6+7+8+9+10+11+12+13}{18} = \frac{72}{18} = 4$
a33 = (-16, -14, -12, -10, -8, -6, -4, -2, 2, 3, 4, 5, 6, 7, 8, 9, 20, 26)	$R(a33) = \frac{-16+(-14)+(-12)+(-10)+(-8)+(-6)+(-4)+(-2)+2+3+4+5+6+7+8+9+20+26}{18} = \frac{18}{18} = 1$

Reduced Crisp value problem is given by the following matrix,

		Player B I	II	III
Player A	1	3	1	1
	2	1	1	5
	3	1	4	1

Now Subtracting 2nd row form the 1st row and 3rd row from the 2nd row and write the differences in the form of two successive rows below the matrix,

	Player B		
Player A	3	1	1
	1	1	5
	1	4	1
A1 - A2	2	0	-4
A2 - A3	0	-3	4

Similarly, subtracting 2nd column form the 1st column and 3rd column from the 2nd column and write the differences in the form of two successive columns to the right of the matrix,

	Player B			**B1 - B2**	**B2 - B3**
Player A	3	1	1	2	0
	1	1	5	0	-4
	1	4	1	-3	3
A1 - A2	2	0	-4		
A2 - A3	0	-3	4		

Now we shall calculate the oddments for A's 1,2,3 strategies and B's I, II, III strategies,

Oddment for A's 1st Strategy = det $\begin{vmatrix} 0 & -4 \\ -3 & 3 \end{vmatrix}$ = -12

Oddment for A's 2nd Strategy = det $\begin{vmatrix} 2 & 0 \\ -3 & 3 \end{vmatrix}$ = 6

Oddment for A's 3rd Strategy = det $\begin{vmatrix} 2 & 0 \\ 0 & -4 \end{vmatrix}$ = -8

Oddment for B's Ist Strategy = det $\begin{vmatrix} 0 & -4 \\ -3 & 4 \end{vmatrix}$ = -12

Oddment for B's IInd Strategy = det $\begin{vmatrix} 2 & -4 \\ 0 & 4 \end{vmatrix}$ = 8

Oddment for B's IIIrdStrategy = det $\begin{vmatrix} 2 & 0 \\ 0 & -3 \end{vmatrix}$ = -6

Now we shall write these oddments, neglecting the signs as shown in the table below,

		Player B I	II	III	
Player A	1	3	1	1	12
	2	1	1	5	6
	3	1	4	1	8
		12	8	6	**26**

Now check the sum of oddments of both the players A and B which is same i.e. 26 as shown in the table above. So we can say that both the players A and B uses their all pure strategies and hence the game is solvable by matrix method but if the sum of oddments of both the players is not equal that means both the players do not use their all pure strategies and hence the game is not solvable by matrix method.

Now we shall calculate the optimal strategies of players A and B, by dividing the oddments by the sum of oddments, therefore

$$\textbf{Optimal Strategy of A} = \left(\frac{12}{26}, \frac{6}{26}, \frac{8}{26}\right) \text{ i.e.} \left(\frac{6}{13}, \frac{3}{13}, \frac{4}{13}\right)$$

$$\textbf{Optimal Strategy of B} = \left(\frac{12}{26}, \frac{8}{26}, \frac{6}{26}\right) \text{i.e.} \left(\frac{6}{13}, \frac{4}{13}, \frac{3}{13}\right)$$

$$\textbf{And the Value of game, V} = \frac{3x12 + 1x6 + 1x8}{12+6+8} = \frac{36+6+8}{26} = \frac{50}{26} = \frac{25}{13}$$

Conclusion

- In this paper, Method of Oddments is used for solving the fuzzy matrix game problem.
- In the above example we have considered a fuzzy matrix game problem whose elements are Octadecagonal fuzzy numbers and solved using method of oddments.

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